Midterm II

1. Assuming a production function where output \( y \) was related to the inputs capital \( (x_2) \), labor \( (x_3) \) through the log-log function

\[
\ln(y) = \beta_1 + \beta_2 \ln(x_2) + \beta_3 \ln(x_3) + \epsilon,
\]

and letting \( \beta_2 + \beta_3 = 2 \),

a. (4 points) Show how much the output increases if you double the amount of inputs.

\[
\ln(y') = \beta_1 + \beta_2 \ln(2x_2) + \beta_3 \ln(2x_3) + \epsilon_4 = \beta_1 + \beta_2 \ln(2x_2) + \beta_3 \ln(x_3) + \epsilon_4 + \beta_3 \ln(x_3)
\]

\[
= \frac{\beta_1 + \beta_2 \ln(x_2) + \beta_3 \ln(x_3) + \epsilon_4 + \beta_2 \ln(x_2) + \beta_3 \ln(x_3) + \ln(y)}{\ln(y')}
\]

\[
\ln(y') = \ln(y) + \ln(2) = \ln(y + 2y)
\]

b. (2 points) Show the elasticity of production with respect to capital \( (x_2) \).

\[
e_{x_2} = \frac{dy}{dx_2} \cdot \frac{x_2}{y}
\]

\[
y' = e_{x_2} \cdot \frac{dy}{dx_2} = e^{\beta_2 \ln(x_2)} \cdot \frac{dy}{dx_2} = \beta_2 \cdot \frac{dy}{dx_2} y
\]

\[
e_{x_2} = \beta_2 \cdot \frac{dy}{dx_2} = \beta_2 \cdot \frac{\beta_2}{x_2} y
\]

\[
e_{x_2} = \beta_2 \cdot \frac{1}{x_2} \cdot y
\]

2. (3 points) Using the table provided construct a 95% two-tailed confidence interval for variable A if your estimation output is (degrees of freedom = 60)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.95</td>
<td>0.17</td>
</tr>
</tbody>
</table>

\[
2.95 \pm 2.0 \cdot 0.17 = 2.95 \pm 0.34
\]

\[
261 \leq \beta_2 \leq 3.29
\]
b. (2 points) When would you reject the null hypothesis of A’s coefficient equaling zero if you are certain that the coefficient is positive? Answer using the t-statistics. ($\alpha = 0.05$)

One tailed test $\Rightarrow t_c = 1.671$

You reject the null if your estimate $> 2.95 + 1.671 \cdot 0.37$

$> 3.23$

or $t > 1.671$

c. (1 point) Can you think of another way of rejecting the same null?

when $\text{prob.} < 0.05$

3. a. (2 points) Explain what Type II error is.

Type II is the failure to reject the null when the alternative is true

b. (6 points) Mention 3 ways of decreasing the Type II error.

1) Choose a larger $\alpha$.
2) Increase $T$.
3) Choose an alternative that is not close to hypothesized value.
4. (4 points) What do you use to test for normality of residuals? Explain how this test does it in broad terms?

Jarque-Bera test. It checks if the skewness and kurtosis of your residuals (from your regression) resembles those of a normal distribution.

5. a. (2 points) Why does the least squares estimator in a simple regression have a t-distribution rather than a standard normal?

Because the population variance is unknown, its estimator \( \hat{\sigma}^2 \) has a Chi-squared distribution. You divide the standard normal \( (b_2 - B_2) \) by \( \sqrt{\frac{\hat{\sigma}^2}{\sum (y - \hat{y})^2}} \) and get a t-dist.

b. (2 points) What is the standard error of regression reported in EViews output tables? How is it calculated?

It is the sum of squared residuals divided by its degrees of freedom.
\[
\frac{\sum e^2}{T-k}
\]

c. (2 points) Why is the F-test a one-tailed test?

F-test is the ratio of two \( \chi^2 \) statistics, which are positive (since they're squared) by definition.

d. (2 points) For which condition can we use the F-test and t-test interchangeably?

When testing one restriction.
6. a. (4 points) Mention 2 consequences of having multicollinearity among your independent variables in a multiple regression.

- low t-stats, high F and $R^2$,
- insignificant estimates and large confidence intervals,
- sensitive to addition or deletion of obs.
- bad out-of-sample forecasts

b. (4 points) Explain two ways of identifying (validating) this problem of multicollinearity.

- pairwise correlation
- regress indep. variables on each other
- $R^2$ doesn't fall by much when you throw out a varab
- low t, high F and $R^2$