Structural Estimates of the U.S. Sacrifice Ratio

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This article investigates the statistical properties of the U.S. sacrifice ratio—the cumulative output loss arising from a permanent reduction in inflation. We derive estimates of the sacrifice ratio from three structural vector autoregression models and then conduct a series of simulation exercises to analyze their sampling distribution. We obtain point estimates of the sacrifice ratio that are consistent with results reported in earlier studies. However, the estimates are very imprecise, which we suggest reflects the poor quality of instruments used in estimation. We conclude that the estimates provide a very unreliable guide for assessing the output cost of disinflation policy.

KEY WORDS: Disinflation; Identification; Structural shocks; Vector autoregression.

The successful conduct of monetary policy requires policy makers to both specify a set of objectives for the performance of the economy and understand the effects of policies designed to attain those goals. Stabilizing prices, one of the dual goals of U.S. monetary policy, is no different. It is generally agreed, and documented by Barro (1996) and the papers in Feldstein (1999), that permanently low levels of inflation create long-run benefits for society, increasing the level and possibly the trend growth rate of output. There is also a strong belief that engineering inflation reductions involves short-term costs associated with losses in output. Policy makers’ decisions on the timing and extent of inflation reduction depend on a balancing of the benefits and costs of moving to a new, lower level of inflation, which in the end requires estimates of the size of each.

This study focuses on the size of one of these aspects of disinflation policy for the U.S. over the postwar period. Specifically, we investigate the output cost of disinflation policy, usually referred to as the sacrifice ratio. The sacrifice ratio is the cumulative loss in output, measured as a percent of one year’s gross domestic product (GDP), resulting from a one-percentage-point permanent reduction in inflation.

Although the sacrifice ratio is a key consideration for policy makers, estimating its size is a difficult exercise because it requires the identification of changes in the stance of monetary policy and an evaluation of their impact on the path of output and inflation. Neither one of these determinations is straightforward because it is difficult to gauge the timing and extent of shifts in policy as well as to separate the movements in output and inflation into those that were caused by policy and those that were not. Finally, even if an estimate of the sacrifice ratio can be constructed, it is critical for policy makers to know something about the precision of the estimate.

To investigate the U.S. sacrifice ratio and its statistical properties, we employ structural vector autoregression (VAR) estimation methods. Within this framework, we study models of increasing complexity, beginning with Cecchetti’s (1994) two-variable system, then considering Shapiro and Watson’s (1988) three-variable system, and finally examining Gali’s (1992) four-variable system. In each of these models, we derive estimates of the sacrifice ratio under a different set of identifying restrictions for the structural shocks. To assess the reliability of the sacrifice-ratio estimates, we undertake a series of simulation exercises, based on parametric bootstraps, that allow us to construct confidence intervals.

Our modeling strategy follows the conventional practice in the structural VAR literature of decomposing monetary policy into a systematic and a random component to identify changes in the stance of policy. The systematic component can be thought of as a reaction function and describes the historical response of the monetary authority to movements in a set of key economic variables, while the random component is labeled as “monetary-policy shocks” and signifies actions of the monetary authority that cannot be explained by the policy rule. Using the estimated structural VAR system, we can trace the dynamic responses of variables to a monetary-policy shock and thereby assess the quantitative impact of a shift in policy on output and inflation. Thus, our evaluation of the sacrifice ratio focuses on unanticipated policy shocks, in which the effects of a contractionary policy correspond to a “pure” (exogenous) monetary tightening rather than a systematic (endogenous) response to other shocks.

As an alternative to the use of the structural VAR framework and its focus on the dynamic responses to innovations in the system, one might consider studying the effects of the systematic component of monetary policy. Attempting to measure the sacrifice ratio, and its precision, by looking at periods in which the monetary authority adopted a stronger (or easier) anti-inflationary policy would require that experiments in which there are statistically measurable shifts in policy regime exist in the data. It is our belief that such episodes are not available. This is confirmed by a series of tests for parameter stability, in which we find that the models display little evidence of structural breaks over the sample period we examine. But beyond the practical issue of whether we have an appropriate history in the sample period, there is the problem of finding an econometric technique that would permit...
a straightforward evaluation of the effect of changes in the policy rule on the economy. Because of their reduced-form nature, structural VAR techniques do not allow us to distinguish between the direct effects of shocks and the indirect effects that arise from their eliciting systematic responses by the monetary authority.

We examine quarterly U.S. data over the period 1959-1997 and consider the behavior of the sacrifice ratio over a one- to five-year horizon. The analysis generates estimates of the sacrifice ratio that vary substantially across the three structural VAR models. Specifically, the estimates range from about 1 to nearly 10, suggesting that somewhere between 1% and 10% of one year’s GDP must be sacrificed for inflation to fall one percentage point. Although the point estimates are broadly consistent with the results of previous studies, the analysis also suggests that the sacrifice ratio is very imprecisely estimated. For example, a 90% confidence interval covers 0 for the point estimates in each model. Further, the sacrifice-ratio estimates display even greater imprecision as the models allow for additional structural shocks. Although the simplest two-variable system indicates that the true value of the sacrifice ratio may lie somewhere between 0.5 and 3.8, the four-variable system suggests a possible range that extends from −43 to +68. We do not take these latter estimates too literally because the values seem extremely implausible given our reading of the recent history. Nevertheless, they serve as a caution to policy makers in that the point estimates do not seem to provide a very reliable guide for gauging the output cost of disinflation policy.

After presenting the basic results, we look into the possible sources of the imprecision in the estimates. Our findings do not suggest that the imprecision arises from structural instability of the reduced-form VAR’s or from a deterioration in the forecasting performance of higher-dimensional models. Rather, the evidence points to weak instruments implicitly used to estimate the models. This latter finding is consistent with the conclusions of Pagan and Robertson (1998), who reported substantial randomness in structural VAR estimates of the liquidity effect and documented that poor instrument quality is an important source of the imprecision. Because the models examined in this study and by Pagan and Robertson share similar identification schemes, future research employing the structural VAR methodology may need to be particularly conscious of the issue of instrument quality and its implications for the reliability of the estimates.

1. THE SACRIFICE RATIO

Most people believe that attempts on the part of a monetary authority to lower the inflation rate will lead to a period of increased unemployment and reduced output. The reason disinflationary episodes have this effect on real economic activity is that inflation displays a great deal of persistence or inertia. That is, price inflation [measured by indexes such as the consumer price index (CPI)] tends to move slowly over time, exhibiting very different behavior from things like stock or commodity prices. Thus, the adjustment process during a disinflation requires the monetary authority to slow aggregate demand growth, creating a period of temporary slack in the economy that will lower the inflation rate only eventually.

There are a number of explanations for inflation’s slow adjustment and the absence of costless disinflation. Fuhrer (1995) provided a review of recent discussions and focused on three possibilities. First, inflation persistence may arise from the overlap and nonsynchronization of wage and price contracts in the economy. Because wages and prices adjust at different times, as well as to each other, slowing the process takes time. Second, people’s inflation expectations may adjust slowly over time, being based on a sort of adaptive mechanism. Because decisions about wages and prices depend on expectations of future changes, slow adaptation is self-fulfilling, creating inertia. And third, if people do not believe that the monetary authority is truly committed to reducing inflation, then inflation will not fall as rapidly. That is, the credibility of the policy maker is important in determining the dynamics of inflation, with less credibility leading to more persistence.

The view that reductions in inflation are accompanied by a period of decreased output (relative to trend) has generated considerable debate among economists on how to lessen the costs of disinflation. Some discussions, including those of Okun (1978), Gordon and King (1982), Taylor (1983), Sargent (1983), Schelde-Andersen (1992), and Ball (1994), have focused on the speed of disinflation and whether the monetary authority should adopt a gradualist approach or subject the economy to a “cold turkey” remedy. Other discussions have focused on identifying the sources of inflation persistence and analyzing the implications of the level of inflation (Ball, Mankiw, and Romer 1988), the degree of nominal wage rigidity (Grubb, Jackman, and Layard 1983), and the extent of central bank independence (Jordan 1997) for the pursuit of cost-reducing strategies.

Although the question of how to lessen the costs of disinflation raises a number of important issues, the present analysis does not focus on this debate. Rather, we contend that discussions about the design and implementation of disinflation policy cannot proceed without some understanding of the quantitative impact of monetary policy on output and inflation. Thus, it is our view that the measurement of the sacrifice ratio is a prerequisite to any study attempting to evaluate its key determinants or the impact of alternative policies on the costs of disinflation.

With this practical issue in mind, a number of authors have estimated sacrifice ratios for the United States using a variety of techniques. Okun (1978) examined a family of Phillips curve models and derived estimates that range from 6% to 18% of a year’s gross national product, with a mean of 10%. Gordon and King (1982) used traditional and VAR models to obtain estimates of the sacrifice ratio that range from 0 to 8. Mankiw (1991) examined the 1982-1985 Volcker disinflation and used Okun’s law to arrive at a “back-of-the-envelope” estimate of 2.8. More recently, Ball (1994) examined movements in trend output and trend inflation over various disinflation episodes and obtained estimates that vary from 1.8 to 3.3.

Although the estimates calculated by Ball (1994) and Mankiw (1991) are of roughly the same order of magnitude, suggesting that a consensus may exist about the size of the
sacrifice ratio, there are several issues remaining. First, prior studies do not, in our view, adequately control for the impact of nonmonetary factors on the behavior of output and inflation. Consider the plots of the quarterly growth rate of real GDP and the CPI displayed in Figures 1 and 2. Although some of the movements in output and inflation over the post-World War II period are surely attributable to monetary-policy actions, it is unreasonable on either theoretical grounds or from visual inspection to believe they all are. Thus, computing a meaningful estimate of the sacrifice ratio requires more than simply calculating a measure of the association between output and inflation during arbitrarily selected episodes. Rather, it depends critically on isolating which movements result from monetary influences.

Second, previous studies such as that of Gordon and King (1982) have failed to account for the policy process. Some of the actions undertaken by a monetary authority are intended to accommodate or offset shocks to the economy. However, the analysis of Gordon and King did not allow the movements in a policy variable to be separated into those associated with a shift in policy and those reflecting a systematic response to the state of the economy. This type of decomposition, which is necessary to assess the effects of monetary policy on the economy, requires the specification and estimation of a structural economic model.

Another important issue concerns the periods selected for the empirical analysis. Studies such as that of Ball (1994) have focused solely on specific disinflationary episodes—periods when contractionary monetary policies are thought to have resulted in the reduction in both inflation and output. However, it is not obvious that estimates of the sacrifice ratio should exclude a priori episodes in which inflation and output are both increasing. Such an approach would only be justified if there were an accepted asymmetry in the impact of monetary policy on output and prices. In the absence of such evidence, economic expansions would contain episodes in which output and inflation increased as a result of expansionary monetary policy. Such episodes would then be as informative about the sacrifice ratio as a disinflation.

Filardo (1998) was an exception because he presented evidence that the sacrifice ratio for the United States varies across three regimes corresponding to periods of weak, moderate, and strong output growth. Using a measure of the sacrifice ratio similar to that used in this study, Filardo derived sacrifice-ratio estimates of 5.0 in the weak-growth regime and 2.1 in the strong-growth regime.

Although the findings of Filardo offer new and interesting insights into the output cost of disinflation, his study, as well as previous work, was silent on the accuracy of the estimates. Specifically, there has been no serious attempt to characterize the statistical precision of sacrifice-ratio measures. Estimation of economic relationships and magnitudes inherently involves some uncertainty, and it is extremely important to quantify their reliability. For example, policy makers may be reluctant to undertake certain policy actions unless they can attach a high degree of confidence to the predicted outcomes. Characterizing the precision of sacrifice-ratio estimates is a primary goal of our analysis.

We now turn to a discussion of the structural VAR methodology and a description of the models that we use to construct sacrifice-ratio estimates.

2. STRUCTURAL VECTOR AUTOREgressIONS

2.1 Identifying Monetary-Policy Shocks and Deriving Estimates of the Sacrifice Ratio

The structural VAR methodology remains a popular technique for analyzing the effects of monetary policy on the economy. A structural VAR can be viewed as a dynamic simultaneous-equations model with identifying restrictions based on economic theory. In particular, the structural VAR relates the observed movements in a variable to a set of structural shocks—innovations that are fundamental in the sense that they have an economic interpretation. In the formulation of their identification assumptions, the models we employ appeal to economic theories that then allow us to interpret one of the structural innovations as a monetary-policy shock. For this reason, we find the structural VAR methodology attractive in evaluating the impact of monetary policy on output and inflation and giving us a measure of the sacrifice ratio. Admittedly, the structural VAR methodology is not without its limitations. The estimated effects of shocks can vary considerably as a result of slight modifications to the identifying restrictions. Thus, it may be quite important to examine a set of models when drawing inferences based on this approach.

Our strategy for deriving an estimate of the sacrifice ratio can be illustrated within a relatively simple system that only includes output and inflation. Following Cecchetti (1994), we consider the following structural VAR model:

\[
\begin{align*}
\Delta y_t &= \sum_{i=1}^{n} b_{1i} \Delta y_{t-i} + \sum_{i=1}^{n} b_{12i} \Delta \pi_{t-i} + \epsilon_t^y \\
\Delta \pi_t &= b_{21} \Delta y_t + \sum_{i=1}^{n} b_{21i} \Delta y_{t-i} + \sum_{i=1}^{n} b_{22i} \Delta \pi_{t-i} + \epsilon_t^n
\end{align*}
\]  

(1)
that can be rewritten more conveniently as

\[ B(L) \begin{bmatrix} \Delta y_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \end{bmatrix}, \]

where \( y_t \) is the log of output at time \( t \), \( \pi_t \) is the inflation rate between time \( t - 1 \) and \( t \), \( \Delta \) denotes the difference operator \((1 - L)\), \( B(L) \equiv [B_{ij}(L)] \) for \( i, j = 1, 2 \) is a \((2 \times 2)\) matrix of polynomial lags, and \( \varepsilon_t = [\varepsilon_t^y, \varepsilon_t^\pi]' \) is a vector innovation process that contains the shocks to aggregate supply \((\varepsilon_t^y)\) and aggregate demand \((\varepsilon_t^\pi)\). It is assumed that \( \varepsilon_t \) has zero mean and is serially uncorrelated with covariance matrix \( E[\varepsilon_t \varepsilon_t'] = \Omega \) for all \( t \). The model includes the change in the inflation rate to allow shocks to have a permanent effect on its level.

Our primary interest is in the impact of the structural shocks on output and inflation over time. To evaluate these magnitudes, we can look at the vector moving average (VMA) representation of \((1)\), which provides the impulse responses of the system to the structural shocks. This is written as

\[
\begin{bmatrix} \Delta y_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{\infty} a_{11}^i \varepsilon_{t-i} + \sum_{i=0}^{\infty} a_{12}^i \varepsilon_{t-i}^\pi \\ \sum_{i=0}^{\infty} d_{21}^i \varepsilon_{t-i} + \sum_{i=0}^{\infty} d_{22}^i \varepsilon_{t-i}^\pi \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \end{bmatrix}. \tag{3}
\]

If we initially use aggregate demand shocks to identify shifts in monetary policy, then \((3)\) provides a particularly convenient representation to assess the dynamic impact of a monetary policy shock on output and inflation. An estimate of the sacrifice ratio can then be computed based on the structural impulse response functions from \((3)\).

For inflation, the sum of the first \( \tau \) coefficients in \( A_{22}(L) \) measures the effect of a monetary-policy shock on its level \( \tau \) periods forward. In the case of output, however, the sacrifice ratio requires us to consider the cumulative effect of a monetary policy shock on output and inflation. An estimate of the sacrifice ratio can then be computed based on the structural impulse response functions from \((3)\).

One set of identifying restrictions is based on the assumption that the structural shocks are uncorrelated and have unit variance. That is, \( \Omega = I_n \), where \( I_n \) is the \((n \times n)\) identity matrix. In addition to assuming that \( \Omega \) is the identity matrix, there are three other sets of identifying restrictions that are employed in the estimation of \( A_0 \). Two of these sets focus on the effects of structural shocks on particular variables and involve short-run restrictions \((A_0)\) restrictions) or long-run restrictions \([A(1)\) restrictions]. The third involves restrictions on the coefficients of contemporaneous variables in the structural equations \((B_0\) restrictions).

Following Blanchard and Quah (1989), our additional identifying restriction for the Cecchetti model is that aggregate demand shocks have no permanent effect on the level of output. This is equivalent to the condition that \( A_{12}(1) = \sum_{i=0}^{\infty} a_{12}^i = 0 \).

The Cecchetti model only identifies two shocks and associates shifts in monetary policy with the aggregate demand disturbance. Although this identification scheme is useful for illustrating the structural VAR methodology and may provide a good approximation for analyzing the relative importance of nominal and real shocks, the framework could yield
misleading estimates of the sacrifice ratio. Specifically, the restrictions in the Cecchetti model fail to identify separate components of the aggregate demand disturbance. Thus, the estimated monetary-policy shock would not only encompass policy shifts but also other shocks related to government spending or shifts in consumption or investment functions.

To provide a more detailed analysis, we also derive estimates of the sacrifice ratio from models developed by Shapiro and Watson (1988) and Gali (1992). These models allow us to decompose the aggregate demand disturbance into separate components and therefore can be used to judge the sensitivity of the results to alternative measures of the monetary-policy shock.

Drawing on the work of Shapiro and Watson (1988), we consider a three-variable system given by

\[
\begin{bmatrix}
\Delta y_i \\
\Delta \pi_i \\
(i_i - \pi_i)
\end{bmatrix} = A(L) \begin{bmatrix}
\epsilon_i^y \\
\epsilon_i^{LM} \\
\epsilon_i^{IS}
\end{bmatrix},
\]

where \( i_i \) is a short-term nominal interest rate, \((i_i - \pi_i)\) is an ex post real interest rate, and \( A(L) \) is a \((3 \times 3)\) matrix of polynomial lags. Although we refer to this as the "Shapiro–Watson model," our specification actually differs from theirs in two small ways. First, Shapiro and Watson decomposed the aggregate supply disturbance into a technology shock and a labor-supply shock. Second, they also included oil prices as an exogenous regressor in their structural VAR system.

Based on this three-variable system, we are able to identify three structural shocks, where \( \epsilon_i^y \) continues to denote an aggregate supply disturbance and where the aggregate demand disturbance is now decomposed into an LM shock and an IS shock, denoted, respectively, by \( \epsilon_i^{LM} \) and \( \epsilon_i^{IS} \). We identify the structural shocks using both short-run and long-run restrictions. The Blanchard–Quah restriction allows us to identify the aggregate supply shock. In addition, we discriminate between IS and LM shocks by assuming that monetary policy has no contemporaneous effect on output \((a^0 = 0)\). We estimate the sacrifice ratio by identifying monetary-policy shifts with LM shocks.

The Gali (1992) model, which allows for the identification of a fourth structural shock, can be written as

\[
\begin{bmatrix}
\Delta y_i \\
\Delta \pi_i \\
(i_i - \pi_i) \\
(\Delta m_i - \pi_i)
\end{bmatrix} = A(L) \begin{bmatrix}
\epsilon_i^y \\
\epsilon_i^{LM} \\
\epsilon_i^{MD} \\
\epsilon_i^{IS}
\end{bmatrix},
\]

where \( m_i \) is the log of the money supply, \((\Delta m_i - \pi_i)\) is the growth of real money balances, and \( A(L) \) is a \((4 \times 4)\) matrix of polynomial lags.

The structural demand shocks \( \epsilon_i^{MD} \) and \( \epsilon_i^{IS} \) denote a money-supply shock, a money-demand shock, and an IS shock, respectively. The identification of the structural shocks are again based on short-run and long-run restrictions. We retain the Blanchard–Quah restriction and assume that the three aggregate demand shocks have no permanent effect on the level of output. Further, we follow Gali and adopt two additional assumptions. The first is that neither money demand nor money supply affect output contemporaneously \((a^0 = a^0_d = 0)\). The second assumption is that contemporaneous prices do not enter the money-supply rule \((b^0 = b^0_d = 0)\). Our estimate of the sacrifice ratio uses money-supply shocks to identify changes in the stance of monetary policy.

Although the inflation rate does not appear as an individual variable in (12), we can recover its impulse response functions from those estimated for \( \Delta i_i \) and \((i_i - \pi_i)\). Specifically, we can use the following relationship

\[
\pi_i = i_i - (1 - L)^{-1} \Delta i_i - (i_i - \pi_i)
\]

to obtain an estimate of the impact of a monetary-policy shock on the level of inflation \( \tau \) periods forward:

\[
\left( \frac{\partial \pi_{\tau+i}}{\partial \epsilon_{\text{MS}}} \right)_i = \sum_{i=0}^{\tau} a^0_{i+2} - a^0_{i+2}.
\]

### 2.2 Computing Confidence Intervals

The sacrifice-ratio estimates we compute are not the true values but are random variables with distributions. It is natural to compute and report some measure of the precision for our estimates. There are several possible procedures for computing confidence intervals for \( \tilde{S}_e(\tau) \). One possibility would be to note that the sacrifice ratio is a function of the parameters of the structural VMA and use the delta method. This is computationally demanding and infeasible for the more complex models we study. Instead we employ simulation-based methods to construct exact small-sample distributions of the estimator of the sacrifice ratio. The procedure can be briefly described as follows.

Let \( \theta \) denote the vector of parameters that constitute the reduced-form VAR model, and let \( \hat{\theta} \) and \( \hat{\Sigma} \) denote, respectively, the estimated parameter vector and estimated variance-covariance matrix of the residuals of the reduced-form VAR using the observed set of data. Our simulation approach is based on specific distributional assumptions about \((\hat{\theta}, \hat{\Sigma})\) and follows the procedure described by Doan (1992, chap. 10). Specifically, we construct a sequence of random draws of \( \Sigma \) from an inverted Wishart distribution, each of which is then used to generate a draw for \( \theta \). For each draw of \((\theta^i, \Sigma^i)\), we can impose the relevant identification restrictions and estimate an artificial structural VAR model, compute its impulse response functions, and then construct an estimate of the sacrifice ratio denoted by \( S_i(\theta^i, \Sigma^i) \). In this manner, a series of \( N \) simulations can be undertaken and used to construct \( N \) estimates of the sacrifice ratio denoted by \( S_i(\theta^i, \Sigma^i), S_j(\theta^j, \Sigma^j), \ldots, S_n(\theta^n, \Sigma^n) \). We can then construct confidence intervals for \( S_e(\tau) \) based on the range that includes the specified percent of the values for \( S_i(\theta^i, \Sigma^i) \).

Simulation methods can also provide insights into the presence of bias in the point estimates. Specifically, we can use information from the simulations to report median bias-corrected point estimates of the sacrifice ratio using the following formula:

\[
\tilde{S}_e(\tau) = \tilde{S}_e(\tau) - \left[ S_e(\tau) - \tilde{S}_e(\tau) \right],
\]

where \( \tilde{S}_e(\tau) \) is the bias-adjusted point estimate of the sacrifice ratio, \( S_e(\tau) \) is the sacrifice-ratio estimate from the structural VAR model, and \( S_e^{0.5}(\tau) \) is the median sacrifice-ratio...
estimate the imprecision associated with the sacrifice-ratio estimates. A similar procedure can be used to construct bias-adjusted estimates of the impulse response functions and confidence intervals.

As a final note, one could argue that our results may actually understimate the imprecision associated with the sacrifice-ratio estimates. The analysis abstracts from real-time data considerations and its implications for the conduct of monetary policy. For example, variables such as output (or unemployment rate) gaps are typically constructed and used by the monetary authority for forecasting purposes or in Taylor-type rules to gauge policy actions. Because these variables are unobserved and very imprecisely measured (Orphanides 2000a,b), their inclusion in the models we study would only lead to greater unreliability in the sacrifice-ratio estimates.

The analysis now turns to a presentation of the results and a discussion of the point estimates of the sacrifice ratio and their associated confidence intervals.

3. EMPIRICAL RESULTS

We construct sacrifice-ratio estimates from the three structural VAR models using quarterly data over the sample period 1959:Q1–1997:Q4. Output is measured by real GDP and inflation is measured by the CPI. The sacrifice-ratio estimates are based on a different transformation of the data for output and prices. Output growth is measured at a quarterly rate in percentage terms, while inflation is measured at an annual rate in percentage terms. The short-term interest rate represents the yield on three-month Treasury bills and the monetary aggregate is measured by M1. The appendix describes the data in further detail.

It is worth noting that preliminary analysis of the data supports the specification of the models. In particular, we examined the stationarity properties of the various series to determine their degree of integration and the presence of cointegrating relationships. The results from the application of Dickey–Fuller (1981) tests provided evidence that $y$, $\pi$, $i$, and $\Delta m$ contain a unit root, but that $(i-\pi)$ and $(\Delta m-\pi)$ are stationary variables. The findings that both output and inflation contain a unit root are particularly important for the identification schemes and the concept of a sacrifice ratio. The evidence of a unit root in the output process allows the long-run restriction on the effects of aggregate demand shocks to be well defined and meaningful, while the evidence of a unit root in the inflation process allows for permanent shifts in its level.

The lag length of the reduced-form VAR was set equal to 8 for the Cecchetti model and equal to 4 for the Shapiro–Watson and Gali models. Estimation of the sacrifice ratio also requires the selection of a horizon for the long-run restriction on aggregate demand shocks. Following Cecchetti (1994), we assume that aggregate demand shocks completely die out after 20 years and truncate the structural VMA representations at 80 quarters.

Table 1 presents the point estimates of the sacrifice ratio at horizons of one to five years, and Figure 3 displays the (median) bias-adjusted estimated responses of output and inflation to a one-unit monetary policy shock across the three models, together with 90% confidence bands. The point estimates should be interpreted as the cumulative output loss corresponding to a permanent one-percentage-point decline in the rate of inflation measured on an annual basis.

The Cecchetti model yields sacrifice-ratio estimates that are relatively constant as the horizon grows, while the other two models show a clear upward trend, with higher output loss at longer horizons. Interestingly, the 20-quarter horizon estimates from the Cecchetti and Shapiro–Watson models, 1.38 and 1.28, respectively, are very similar to the values of 1.8 and 1.4 calculated by Ball (1994) and Schelde-Anderson (1992) for the Volcker disinflation. In contrast, the Gali model generates a markedly higher estimate of 9.87, which is much closer to the mean value obtained by Okun (1978).

An examination of the bias-adjusted impulse response functions in Figure 3 reveals a pattern that is qualitatively similar across models and accords with the predicted effects of a monetary tightening. Output declines in response to a monetary-policy shock before eventually returning to its initial level. (The nature of the identification restrictions in the Shapiro–Watson and Gali models precludes a contemporaneous response of output to the monetary-policy shock.) Inflation also decreases and displays a permanent decline in response to the monetary-policy shock.

As shown, there are some differences in the extent and size of the impact of the monetary-policy shock across models. For example, most of the response of output and inflation in the Cecchetti model occurs within the first few quarters after the shock. Relative to the Cecchetti and Shapiro–Watson models, the Gali model suggests a deeper and more protracted output decline along with a smaller decrease in inflation. Both of these effects lead to the higher point estimates of the sacrifice ratio that emerge from this model.

Figure 3 also indicates that the impulse response functions are not estimated very precisely. Because of the nature of the long-run identifying restrictions, one would expect the confidence intervals for output to narrow and converge around 0 as the horizon steadily increases. However, the results indicate that the confidence intervals at shorter horizons typically cover 0 and can be fairly wide. In the case of inflation, the confidence intervals for the Shapiro–Watson and Gali models seem particularly wide and either cover or lie very close to 0. An initial reading of this evidence reveals the imprecision associated with the quantities of interest and clearly hints at the potential difficulty of obtaining reliable estimates of the sacrifice ratio.

Table 2 presents the results for the simulated distributions of the sacrifice ratio based on the procedures outlined in the previous section, with 10,000 replications. We report (median)
bias-adjusted estimates of the sacrifice ratio and 90% confidence intervals for each horizon. The empirical density functions for $S_e(\tau = 20)$ are plotted in Figure 4.

There are several striking results that emerge from Table 2 and Figure 4. The first is that a 90% confidence interval for the sacrifice ratio includes 0 for all three models. That is, we cannot with any reasonable degree of certainty rule out the possibility that $S_e(\tau = 0)$ at each horizon. Second, the Shapiro–Watson and Gali models display a marked increase in the imprecision of the sacrifice-ratio estimates as the horizon lengthens. Last, the results are sensitive to the measure of monetary-policy shocks. In particular, expanding the
two-variable system to identify separate components of the aggregate demand disturbance yields ranges for the sacrifice ratio that are highly variable and implausible. Taken together, these findings speak directly to the unreliability of the point estimates and suggest that we have little understanding about the quantitative impact of monetary policy on output and inflation.

Looking at Figure 4, we see that the simulated distribution of the sacrifice ratio for the Cecchetti model is non-normal, although it is reasonably symmetric. In addition, the bias-corrected bootstrapped estimates of the sacrifice ratio are nearly identical with the original estimates, providing little evidence of bias. For the Shapiro–Watson and Gali models, however, the simulated distributions of the sacrifice ratio reveal a markedly different picture. The distributions are non-normal, asymmetric, and extremely long tailed. Further, there is now increased evidence of bias in the point estimates from both models.

To explore the behavior of the sacrifice-ratio estimates in further detail, we examined the simulated distribution of the estimators of the impact responses of output and inflation to a monetary-policy shock. Because the features of the estimators for each model were broadly similar across horizons, we only display and discuss the results for a horizon of \( \tau = 20 \) quarters. Figure 5 plots the corresponding empirical density functions. The graphs relate to the relevant quantities in the numerator and denominator of the sacrifice ratio and therefore provide estimates of the cumulative response of the level of output and the response of the level of inflation to a monetary-policy shock.

The impact response estimators have very non-normal distributions that become distinctly bimodal as the number of structural shocks increases. In addition, the simulated distributions of \( \Sigma \hat{\alpha}_{12} \) do not restrict the output response to be negative and display greater variability as the complexity of the models increases. In the case of the response of inflation to a monetary-policy shock, the estimated sampling distribution of \( \Sigma \hat{\alpha}_{12} \) lies exclusively below 0 for the Cecchetti model, while the corresponding sampling distributions for the Shapiro–Watson and Gali models display a pronounced rightward shift and are centered near 0. This latter result would seem to account for the extremely wide confidence bands associated with the sacrifice-ratio estimates from the Shapiro–Watson and Gali models. Because the estimated inflation response appears in the denominator of the expression for the sacrifice ratio, values of this magnitude close to 0 will lead, for a given output response, to larger (absolute) estimates of the sacrifice ratio. The Shapiro–Watson and Gali models also seem capable of generating positive estimates of the sacrifice ratio in the presence of perverse output and inflation responses.

4. EXPLORING THE SOURCES OF THE IMPRECISION

There are a number of possible sources for the imprecision we find in the estimates of the sacrifice ratio. In this section we examine three. First, we look at the possibility that the estimated VAR’s contain structural breaks. If they do, then the wide confidence bands we obtain could reflect specification error arising from the estimation of a time-invariant model. We examine this possibility with a set of structural break tests based on the work of Andrews (1993) and Andrews and Ploberger (1994). Much to our surprise, we are able to conclude that all three of the models we analyze are stable over the sample period we study.

<p>| Table 2. Median Bias-Adjusted Estimates and Confidence Intervals for Sacrifice Ratio; Simulation Experiment Based on 10,000 Replications |
|----------------------------------------|----------------------------------------|----------------------------------------|</p>
<table>
<thead>
<tr>
<th>Horizon (quarters)</th>
<th>Median bias-adjusted estimate</th>
<th>Median bias-adjusted 90% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cecchetti model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 4 )</td>
<td>1.3072</td>
<td>(0.0110, 2.4686)</td>
</tr>
<tr>
<td>( \tau = 8 )</td>
<td>1.3203</td>
<td>(-0.3782, 2.6474)</td>
</tr>
<tr>
<td>( \tau = 12 )</td>
<td>1.5806</td>
<td>(-0.5048, 3.8141)</td>
</tr>
<tr>
<td>( \tau = 16 )</td>
<td>1.5482</td>
<td>(-0.3341, 3.9162)</td>
</tr>
<tr>
<td>( \tau = 20 )</td>
<td>1.3591</td>
<td>(-0.4200, 3.2170)</td>
</tr>
<tr>
<td>Shapiro–Watson model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 4 )</td>
<td>0.4131</td>
<td>(-2.4175, 4.5281)</td>
</tr>
<tr>
<td>( \tau = 8 )</td>
<td>2.8049</td>
<td>(-11.5053, 18.6673)</td>
</tr>
<tr>
<td>( \tau = 12 )</td>
<td>3.4605</td>
<td>(-22.7007, 30.0720)</td>
</tr>
<tr>
<td>( \tau = 16 )</td>
<td>3.9607</td>
<td>(-27.6783, 36.4351)</td>
</tr>
<tr>
<td>( \tau = 20 )</td>
<td>4.3495</td>
<td>(-30.2247, 43.0066)</td>
</tr>
<tr>
<td>Gali model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 4 )</td>
<td>1.0911</td>
<td>(-2.4804, 5.2475)</td>
</tr>
<tr>
<td>( \tau = 8 )</td>
<td>4.9281</td>
<td>(-13.4865, 25.3286)</td>
</tr>
<tr>
<td>( \tau = 12 )</td>
<td>8.2839</td>
<td>(-34.1267, 48.7030)</td>
</tr>
<tr>
<td>( \tau = 16 )</td>
<td>11.2572</td>
<td>(-41.9125, 65.5307)</td>
</tr>
<tr>
<td>( \tau = 20 )</td>
<td>14.0334</td>
<td>(-43.4513, 67.9279)</td>
</tr>
</tbody>
</table>

Figure 4. Empirical Density Functions for Sacrifice-Ratio Estimates.
Second, we ask whether the results reflect the fact that the higher-dimensional models provide worse forecasts of output and inflation. If they do, then it may not be the addition of more structural shocks that leads to less precise estimates of the sacrifice ratio. Instead, it may be that the Shapiro–Watson and Gali models are just poorer characterizations of the data. To address this issue, we compare the out-of-sample forecasting performance of each of these models for output and
inflation. The findings indicate that the more complex models provide better, not worse, forecasts, so this is not the explanation for our results.

Finally, we look at the issue of instrument quality. Estimation of structural VAR models can be viewed in an instrumental-variables setting. The use of weak instruments could lead to imprecision in the calculation of the elements that go into the computation of the sacrifice ratio. We provide some evidence here that this is a likely source of at least part of the imprecision in the sacrifice-ratio estimates.

Before we proceed to examine each of these three possible sources of imprecision in detail, we would like to comment on one avenue we do not pursue. Specifically, we do not attempt to provide a detailed investigation into the relative contributions of $A_0$, the matrix of the contemporaneous effects of the structural disturbances on the endogenous variables, and $C(L)$, the reduced-form VMA representation, to the variability of the sacrifice-ratio estimates. The most natural way to attempt to parse the relative impact of these on the imprecision of the sacrifice ratio would be to hold one of these two quantities fixed and then use simulated data to estimate the variability induced by the other. Unfortunately, this approach is not feasible because the value of $A_0$ depends on $C(L)$ through the identifying restrictions, so it cannot be held constant while new data are generated to reestimate $C(L)$.

4.1 Structural Breaks

The results in Section 3 are estimated over a nearly 40-year period beginning in 1959. Many things may have changed in the United States over this time, not the least of which is Federal Reserve policy. Each of our models includes an explicit or implicit monetary-policy reaction function and, if the policy regime were to change, then one would expect the coefficients in these equations to change as well. In other words, we have every reason to believe that the models we estimate may contain structural breaks.

To investigate this issue, we use tests proposed by Andrews (1993) and Andrews and Ploberger (1994) to study the possibility of a break in the specification of the models. Table 3 reports the results of the three commonly used tests for structural change, all of which are Lagrange multiplier (LM) tests and allow for an endogenous determination of the break date. We report the value of the LM test statistics, together with their $p$ values, for each equation in each of the three models. The results are quite striking. It is only the Andrews–Ploberger average LM test that provides evidence of a structural break for one of the equations in the Stock–Watson model and two of the equations in the Gali model. Even for these cases, however, the other two tests fail to reject structural stability. From this we conclude that instability of the VAR’s is unlikely to be the source of our results.

4.2 Forecasting Accuracy

If the two-variable system is just a better model than the three- or four-variable systems, then the explanation for our results would be straightforward. The obvious dimension in which to compare models of this type is to examine their forecasting ability. We do this by calculating the accuracy of out-of-sample forecasts for output and inflation using the relevant reduced-form equations (or their equivalents) for each of the three models.

### Table 3. Structural Break Tests: 1959:Q1–1997:Q4

<table>
<thead>
<tr>
<th>Model/equation</th>
<th>Andrews/Quandt</th>
<th>Andrews/Ploberger</th>
<th>Andrews/Ploberger</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>supremum LM test</td>
<td>exponential LM test</td>
<td>average LM test</td>
</tr>
<tr>
<td><strong>Cecchetti model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output equation</td>
<td>23.180 (0.9199)</td>
<td>8.981 (0.8295)</td>
<td>16.612 (0.5084)</td>
</tr>
<tr>
<td>Inflation equation</td>
<td>27.999 (0.6211)</td>
<td>10.626 (0.5555)</td>
<td>18.851 (0.2631)</td>
</tr>
<tr>
<td><strong>Stock–Watson model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output equation</td>
<td>15.455 (0.9936)</td>
<td>6.276 (0.8999)</td>
<td>11.876 (0.6187)</td>
</tr>
<tr>
<td>Inflation equation</td>
<td>22.186 (0.6616)</td>
<td>8.841 (0.4274)</td>
<td>15.521 (0.1790)</td>
</tr>
<tr>
<td>Ex post real interest-rate equation</td>
<td>24.579 (0.4600)</td>
<td>10.233 (0.2225)</td>
<td>18.722 (0.0372)</td>
</tr>
<tr>
<td><strong>Gali model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output equation</td>
<td>30.524 (0.4320)</td>
<td>12.206 (0.3081)</td>
<td>20.649 (0.1337)</td>
</tr>
<tr>
<td>Nominal interest-rate equation</td>
<td>29.371 (0.5166)</td>
<td>12.135 (0.3176)</td>
<td>19.530 (0.2067)</td>
</tr>
<tr>
<td>Ex post real interest-rate equation</td>
<td>27.114 (0.6877)</td>
<td>12.123 (0.3192)</td>
<td>22.524 (0.0586)</td>
</tr>
<tr>
<td>Real money balances equation</td>
<td>34.415 (0.2041)</td>
<td>13.860 (0.1404)</td>
<td>23.910 (0.0298)</td>
</tr>
</tbody>
</table>

**NOTE:** Structural stability tests correspond to the joint test for constancy of all regression parameters in a particular equation. Values of the test statistic are reported in columns 2–4, with asymptotic $p$ values reported below in parentheses.
Table 4 reports the results of an exercise in which we estimate each model, generate one-quarter-ahead forecasts, add an additional quarter’s worth of data, and repeat the exercise. This expanding-sample estimation begins with data from 1959:Q1 through 1975:Q1 and rolls forward through 1997:Q4. We report the root mean squared error of these one-step-ahead out-of-sample forecasts. The conclusion is clear. As we increase the number of variables in the system, the models improve at forecasting output growth. The accuracy of the inflation forecasts is roughly equivalent, with the two-variable system performing worst. The last result is not a surprise because adding information rarely improves inflation forecasts. But overall, we do not find evidence indicating that the forecasting performance deteriorates as the complexity of the model increases.

4.3 Instrument Quality

Pagan and Robertson (1998) recently showed that structural VAR models can be cast in a generalized method of moments framework and that the restrictions used to identify the structural shocks generate instruments for the estimation of $A_0$. In addition, they found that structural VAR estimates of the liquidity effect are extremely imprecise due in large part to the poor quality of instruments used in estimation. The results of Pagan and Robertson suggest that instrument quality may be a relevant consideration for our results.

Following the approach of Pagan and Robertson, we adopt the testing procedure outlined by Shea (1997) to evaluate the quality of instruments used in the estimation of the three structural VAR models. An advantage of this methodology is that it can be applied to equations with multiple endogenous explanatory variables. The testing procedure yields a partial $R^2$ measure indicating the extent to which the instrument set has components important to an endogenous variable that are independent of those important to the other endogenous variables.

Table 5 reports the partial $R^2$ measure for the instruments used in estimating the structural equations. Although we are able to examine the quality of instruments for each equation, the results for the output equation and inflation equation in the Cecchetti and Shapiro–Watson models are of particular interest. In the case of the Gali model, we will focus our attention on the output equation as well as on the nominal and real interest-rate equations.

Across all three models, the long-run restrictions yield instruments of relatively low quality for estimating the coefficients of the output (aggregate supply) equation. This is especially true in the case of the Gali model, in which the instruments seem to be particularly poor. For the inflation equation, or the nominal and real interest-rate equations in the Gali model, there is evidence of some improvement in the quality of the instruments. However, some caution may be needed in interpreting the high observed values of the partial $R^2$ measure. As Pagan and Robertson noted, the short-run restrictions used for identification in these latter equations generate instruments that are (structural and reduced-form) residuals from previously estimated equations. Although these variables are assumed to be orthogonal to the structural errors, the high partial $R^2$ measure indicates that they may not be valid instruments and suggests the presence of additional bias in the instrumental-variable estimates. Taken together, the evidence of weak or invalid instruments speaks directly to the tenuous nature of the identifying assumptions and the limited ability of current empirical methodologies to allow researchers to draw reliable inferences about structural economic relationships.

<table>
<thead>
<tr>
<th>Model</th>
<th>Output growth</th>
<th>(Level of) Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cecchetti</td>
<td>0.7348</td>
<td>1.3943</td>
</tr>
<tr>
<td>Shapiro–Watson</td>
<td>0.6856</td>
<td>1.3666</td>
</tr>
<tr>
<td>Gali</td>
<td>0.7072</td>
<td>1.2313</td>
</tr>
</tbody>
</table>

Table 5. Partial $R^2$ Measure for Instruments in the Structural VAR Models

<table>
<thead>
<tr>
<th>Cecchetti model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous variables</td>
</tr>
<tr>
<td>Output equation</td>
</tr>
<tr>
<td>Inflation equation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shapiro–Watson model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous variables</td>
</tr>
<tr>
<td>Output equation</td>
</tr>
<tr>
<td>Inflation equation</td>
</tr>
<tr>
<td>Real interest-rate equation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gali model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous variables</td>
</tr>
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<td>Output equation</td>
</tr>
<tr>
<td>Nominal interest-rate equation</td>
</tr>
<tr>
<td>Real interest-rate equation</td>
</tr>
<tr>
<td>Money growth equation</td>
</tr>
</tbody>
</table>
5. CONCLUSION

This study examines the output cost of disinflation policy for the United States over the postwar period. Across various models, the estimates of the sacrifice ratio imply that a permanent one-percentage-point reduction in inflation entails a loss of approximately 1–10% of a year’s real GDP. The confidence intervals around the point estimates, however, indicate that none of the point estimates differ from 0 at conventional levels of statistical significance. Further, the high degree of imprecision associated with the estimates suggests that our knowledge about the actual impact of monetary policy on the behavior of the economy is quite limited.

The evidence from this study supports the view that identifying restrictions used in estimation are tenuous and generate weak or invalid instruments. Alternative identification schemes are unlikely to change this outcome. Unlike standard situations that allow for the application of instrumental-variables procedures, the instrument set available for the estimation of structural VAR models is very restricted. This lack of valid instruments imposes severe restrictions on the nature of the structural shocks that can be identified. Thus although a better understanding of the true costs of disinflation would be of particular interest and importance to policy makers, we are skeptical that current data and econometric techniques can provide a meaningful set of estimates.

ACKNOWLEDGMENTS

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APPENDIX: DATA

This appendix discusses the data used in the estimation. All data are quarterly. The estimates are conducted over the sample period 1959:Q1–1997:Q4.

Output. The output series GDPH is measured as real GDP in chain-weighted 1992 dollars.

Prices. The price data are a quarterly average of the monthly consumer price series PCU for all urban consumers.

Interest Rates. The interest-rate data are a quarterly average of the monthly series FTBS3, which represents the yield on three-month Treasury bills.

Money Stock. The data on the money stock are for M1 and are a quarterly average of the series FM1. The measures of M1 prior to 1959 are taken from the Federal Reserve Bulletin.

REFERENCES