

# *Purchasing power parity: a cointegration analysis of the Australian, New Zealand and Singaporean currencies*

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This paper provides a brief overview of tests for stationarity and cointegration and applies them in an attempt to establish the validity of purchasing power parity theory for the Australian, New Zealand and Singaporean currencies using quarterly time series for the period 1973 to 1992.

## I. INTRODUCTION

Purchasing power parity theory (PPP) has a long history but really came to the fore during discussions concerning appropriate exchange rates in which countries should rejoin the Gold Standard after the first World War. The theory still commands considerable respect in certain quarters and some financial institutions, such as the Swiss Bank Corporation (see Holzer, 1992), regard it as a useful guide to long-term currency movements.

In recent times, there has been an explosion of empirical research on the validity of PPP in the real world. As Rush and Husted (1985) have pointed out, these studies may be categorized according to whether:

- (i) price and exchange rate levels (absolute PPP) or changes in prices and exchange rates (relative PPP) are studied;
- (ii) individual commodity prices or national price levels are employed;
- (iii) purely traded goods' prices or non-traded as well as traded goods' prices are considered;
- (iv) a bilateral or multilateral approach is adopted;
- (v) the short run or long run is investigated.

In general, such empirical work has tended to yield contradictory results depending upon:

- (i) the particular currencies under consideration;

- (ii) the price indices used to measure price levels or inflation;
- (iii) the particular time period under investigation;
- (iv) the method of analysis employed.

This present study seeks to make a modest contribution to this literature by presenting tests of cointegration between price levels and exchange rates along with tests for stationarity of real exchange rates for Australia, New Zealand and Singapore using quarterly data for the period from 1973 to 1992. Recent papers using this methodology include those by Baillie and Selover (1987), Corbae and Ouliaris (1988), McNown and Wallace (1989), Yoonbai Kim (1990) and Conejo and Shields (1993).

## II. STATIONARITY, INTEGRATED SERIES AND COINTEGRATION: AN OVERVIEW

A stochastic process is said to be *stationary* if the means and variances of the process are constant over time while the autocovariance between two periods depends only on the gap between those periods and not the actual time at which the covariance is considered. If one or more of these three conditions are not met, the process is nonstationary. Intuitively, if a time series, defined as a single realization of a stochastic process, has a tendency to return to a fixed value over time, then it is stationary.

If a time series  $X_t$  has to be differenced  $d$  times in order to induce

stationarity, it is said to be integrated of order  $d$ , denoted  $X_t \sim I(d)$ . A series that requires no such differencing to obtain stationarity is  $I(0)$ . Thus, time series may be classified according to their order of integration. Roughly speaking,  $I(2)$  series appear to be growing at an increasing rate,  $I(1)$  series appear to grow at a constant rate while  $I(0)$  series appear to be trendless.

If two time series  $Y_t$  and  $X_t$  are integrated of different order, say  $I(2)$  and  $I(1)$  respectively, then they must be drifting apart over time. Therefore a regression of  $Y_t$  on  $X_t$  would be illogical and, in any case, the error term produced,  $e_t$ , would also be  $I(2)$  so violating the underlying assumptions of ordinary least squares (OLS). In other words, the linear combination  $Y_t - a - bX_t = e_t$  is  $I(2)$ . In general, if two series are integrated of different orders, linear combinations of them are integrated to the higher of the two orders.

Similarly, if the two series  $Y_t$  and  $X_t$  are both  $I(1)$ , then it is normally the case that a linear combination of them will also be  $I(1)$  so that a regression of one upon the other would produce spurious results. That is, because  $e_t$  is again  $I(1)$ , it still violates the assumptions of OLS. However, in some instances, a linear combination of two  $I(1)$  variables will result in a variable which is  $I(0)$  and, in such cases, the two series are said to be *cointegrated*. Now a regression of  $Y_t$  on  $X_t$  is permissible since the error term  $e_t$  is  $I(0)$  or stationary, so satisfying the assumptions of OLS.

If  $Y_t$  and  $X_t$  are cointegrated, they must move together in the long run and the single regression equation  $Y_t = a + bX_t + e_t$  will represent the cointegrating relationship between  $Y_t$  and  $X_t$  provided the error term  $e_t$  contains no long-run or trend components. At least one of the variables in the cointegrating relationship must move in the short run so as to maintain the long-run relationship. If not, deviations from the long-run relationship would never be corrected and the relationship could not hold in the long run. If, at any time,  $Y_t$  exceeds  $a + bX_t$  then eventually  $Y$  must fall or  $X_t$  must rise or both to maintain the long-run relationship. Therefore, a necessary condition for a long-run equilibrium relationship between  $Y_t$  and  $X_t$  is that they are cointegrated.

### III. TESTING FOR STATIONARITY: DICKEY FULLER (DF) AND AUGMENTED DICKEY FULLER (ADF) TESTS

Consider the following model:

$$Y_t = a + bt + cY_{t-1} + e_t \quad (1)$$

where  $e_t \sim N(0, \sigma^2)$  so that  $e_t \sim I(0)$

It can be shown by repeated substitutions that if  $c = 1$  then, although  $e_t$  has constant variance, the variance of  $Y_t$  tends to infinity as time  $t$  increases. Thus  $Y_t$  is nonstationary and is described as a time series with stochastic trend. Furthermore  $Y_t$  is an integrated series in the sense that the random shocks  $e_t$  accumulate, add up or integrate over time so that every shock affects all later values of  $Y_t$  and the process has an infinite memory. Similarly, if  $b \neq 0$  then  $Y_t$  is itself a specific function

of time  $t$  and is said to have a deterministic trend. In both cases, because  $Y_t$  is nonstationary, standard  $t$  tests for an OLS regression are rendered invalid. Therefore to obviate this problem, Equation 1 is reparameterized to first difference form by subtracting  $Y_{t-1}$  from both sides to produce,

$$\Delta Y_t = a + bt + (c-1)Y_{t-1} + e_t \quad (2)$$

This is known as the Dickey–Fuller regression OLS estimation which provides the basis for a set of three tests.

Test 1  $H_0 : c-1 = 0 \text{ or } c = 1$

This null hypothesis is that  $Y_t$  is  $I(1)$  and is rejected if the  $t$  statistic on  $c-1$  has a larger negative value than the critical value for  $\tau$  in Fuller (1976, p. 373). This test, the so-called ‘unit root test’, is however not sufficient to establish stationarity for, if  $b$  is nonzero, the first difference  $\Delta Y_t$  will be time dependent and so  $Y_t$  cannot be  $I(0)$ .

Test 2  $H_0 : c-1 = 0 \text{ or } c = 1 \text{ and } b = 0$

The null hypothesis here is that the  $Y_t$  has a stochastic trend but no deterministic trend and can be tested using the standard  $F$  test. The estimated value of  $F$  is compared with the critical value of  $\Phi_3$  in Dickey and Fuller (1981, p. 1063). If  $F$  is less than  $\Phi_3$  we cannot reject the null hypothesis and so conclude that  $Y_t$  is a random walk. However,  $\Delta Y_t$  is stationary implying that  $Y_t \sim I(1)$ .

Test 3  $H_0 : c-1 = 0 \text{ or } c = 1 \text{ and } b = 0 \text{ and } a = 0$

The null hypothesis here is that  $Y_t$  has a stochastic trend, no deterministic trend and no drift. If the estimated value of  $F$  is less than the critical value of  $\Phi_2$  in Dickey and Fuller (1981, p. 1063), we may conclude that  $Y_t$  is a random walk with no drift. In other words  $Y_t$  is still  $I(1)$ .

To protect against the possibility that  $Y_t$  follows a higher order autoregressive process, lagged values of the dependent variable may be added to the right hand side of Equation 2 to produce,

$$\Delta Y_t = a + bt + (c-1)Y_{t-1} + \sum_{i=1}^n d_i \Delta Y_{t-i} + e_t \quad (3)$$

where  $n$  is selected to ensure white noise residuals in the regression. This is known as the augmented Dickey–Fuller regression and the three tests outlined above are conducted in precisely the same way.

### IV. TESTING FOR COINTEGRATION: THE ENGLE–GRANGER TWO STAGE PROCEDURE

If we have an economic model involving two time series  $Y_t$  and  $X_t$  and if the DF or ADF tests indicate that both are  $I(1)$  then, usually, the linear combination  $Y_t - a - bX_t = e_t$  will also be  $I(1)$ .

However, if we find that  $e_t$  is stationary, i.e.  $I(0)$ , then  $Y_t$  and  $X_t$  are cointegrated. Therefore we may test for cointegration in two steps:

(i) Estimate the so-called cointegrating regression

$$Y_t = a + bX_t + e_t$$

by OLS and collect the residuals  $\hat{e}_t$ .

(ii) Test whether  $\hat{e}_t$  are stationary by forming the equation

$$\hat{e}_t = \rho \hat{e}_{t-1} + v_t \quad v_t \sim N(0, \sigma^2)$$

and reparameterize to

$$\Delta \hat{e}_t = (\rho - 1) \hat{e}_{t-1} + v_t$$

The null hypothesis  $H_0 : \rho - 1 = 0$  or  $\rho = 1$  may be tested by comparing the calculated  $t$  values against critical values in Engle and Yoo (1987, p. 157). Again, an augmented version is available to cope with higher order autoregression.<sup>1</sup>

## V. PURCHASING POWER PARITY AND COINTEGRATION

Purchasing power parity (PPP) postulates that the exchange rate between two currencies is ultimately determined by the domestic purchasing power of the currencies concerned. In particular, the absolute version of PPP states that, after allowing for transactions costs and in the absence of barriers to trade, the price of a homogeneous basket of commodities in one currency will equal its price in another currency when translated at the current exchange

rate. Were this not so, trader arbitrageurs would purchase goods in the cheaper market and sell in the dearer as long as the price differential exceeded transactions costs. Therefore the absolute version of PPP postulates the long-run equilibrium relationship:

$$D_t = S_t F_t \quad (4)$$

or

$$S_t = (D/F)_t \quad (5)$$

where  $S_t$  is the nominal exchange rate,  $D_t$  is the domestic price level and  $F_t$  is the foreign price level.

For such a long run equilibrium relationship to exist, the two variables  $S_t$  and  $(D/F)_t$  must be cointegrated.

To test for this cointegration, quarterly data for the period 1973–92 on the bilateral exchange rate and the price ratio for Australia, New Zealand and Singapore *vis-à-vis* the USA were obtained from *International Financial Statistics* of the IMF. The price index used was the Consumer Price Index. The test procedure applied was conducted in two stages.

First, the hypothesis that the nominal exchange rate and the ratio of price indices are nonstationary was examined using the three tests outlined in Section III. The results are presented in Table 1. For all three countries, these test results indicate quite clearly that the variables are nonstationary. The DF test and the ADF test, using up to two lags to ensure autocorrelation free residuals, were reapplied to the first differences of each variable (indicated by the symbol  $\Delta$ ) and this time, stationarity was indicated in every instance. It was therefore concluded that all exchange rates and price ratios are nonstationary and  $I(1)$ . Secondly, cointegrating regressions of the form:

$$\log S_t = a + b \log (D/F)_t + e_t$$

were estimated for all three countries and the residuals tested for

Table 1. Tests for unit roots in nominal exchange rate and price ratios

Country	Variable	DF	ADF(1)	ADF(2)	$\Phi_3$	$\Phi_2$
Australia	$S_t$	-1.9660	-1.7937	-1.8466	1.8541	2.5243
	$(D/F)_t$	-1.3761	-1.6507	-1.9652	2.8839	4.6285
	$\Delta S_t$	-9.6280	-7.0882	-4.9521	—	—
	$\Delta(D/F)_t$	-6.8976	-4.2689	-3.8480	—	—
New Zealand	$S_t$	-1.6133	-1.6736	-2.0650	1.7934	2.2848
	$(D/F)_t$	+1.3280	-0.5889	-1.2022	1.7906	3.3106
	$\Delta S_t$	-8.6166	-7.0537	-4.6413	—	—
	$\Delta(D/F)_t$	-4.5228	-3.4160	-3.6343	—	—
Singapore	$S_t$	-2.4322	-2.4448	-1.6872	3.4505	3.0570
	$(D/F)_t$	-2.5783	-2.6559	-2.4861	3.5381	3.3470
	$\Delta S_t$	-9.4043	-8.6391	-6.3127	—	—
	$\Delta(D/F)_t$	-6.0888	-5.5913	-6.0490	—	—
Critical values (0.05)						
(a) Levels		-3.4666	-3.4673	-3.4681	6.49 to 6.73	4.88 to 5.13
(b) First differences		-3.4673	-3.4681	-3.4688	—	—

<sup>1</sup> The interested reader will find a full, yet readily accessible, account of cointegration analysis in Charemza and Deadman (1992).

Table 2. *Cointegrating regressions: nominal exchange rates on price ratios*

Country	a	b	$R_2$	DF	ADF(1)	AD(2)	ADF(3)
Australia	0.2249	1.5064	0.80	-1.97	-1.78	-1.61	-1.88
New Zealand	0.1576	0.9788	0.81	-1.87	-1.93	-2.14	-2.53
Singapore	0.9052	0.5554	0.72	-1.90	-1.63	-0.73	-0.77
Critical values (0.05)				-3.41	-3.42	-3.42	-3.42

stationarity using the DF and ADF procedures. The results are shown in Table 2. In no case is stationarity of the residuals apparent so that cointegration of nominal exchange rates and price ratios cannot be accepted. These results therefore do not lend weight to PPP for the currencies and time period studied.<sup>2</sup>

## VI. THE REAL EXCHANGE RATE

The real exchange rate,  $R_t$ , is a measure of a country's competitiveness *vis-à-vis* another country and may be expressed as:

$$R_t = S_t (F/D)_t \quad (6)$$

If foreign prices  $F_t$  rise above domestic prices  $D_t$  and this is not offset by a depreciation of the foreign currency  $S_t$ , then there is a real appreciation of the foreign currency and the foreign country concerned becomes relatively uncompetitive.

If PPP holds so that  $S_t = (D/F)_t$ , then, in principle, the value of  $R_t$  ought to be unity. Alternatively, taking logs of both sides of Equation 6,

$$\log R_t = S_t + \log F_t - \log D_t \quad (7)$$

and, if  $R_t = 1$  then  $\log R_t = 0$

At any point in time, of course, it is unlikely that  $\log R_t$  will be exactly equal to zero but short run deviations from PPP imply that  $\log R_t$  should be a zero mean stationary process, i.e. I(0). Therefore if  $\log R_t$  is I(1), there is a tendency for the nominal exchange rate and the price ratio to drift apart.

Table 3. *Tests for unit roots in real exchange rates*

Country	DF	ADF(1)	ADF (2)
Australia	-1.9675	-1.7769	-1.6111
New Zealand	-1.8453	-1.9019	-2.1113
Singapore	-2.0136	-1.6513	-0.7314
Critical values (0.05)	-3.4666	-3.4673	-3.4681

Accordingly, the real exchange rates for Australia, New Zealand and Singapore were converted to logarithms as per Equation 7 and tested for unit roots using the DF and ADF tests described previously. The results shown in Table 3 indicate nonstationarity for all three countries and this is entirely consistent with the cointegration tests.

## VII. CONCLUSION

In this paper, evidence that PPP does not hold in the long run for the Australian, New Zealand and Singaporean currencies has been presented. Cointegration tests did not support the existence of a long run equilibrium relation between the consumer price ratio and nominal exchange rate *vis-à-vis* the US dollar for any of the three countries. Similarly, real exchange rates for all three countries were found to be nonstationary processes.

Why is PPP such a poor tool for explaining exchange rate movements? It ignores capital flows. In recent years, in particular, capital has been free to move from country to country. Cross border trade in financial assets swamps foreign exchange transactions in goods and services. It is estimated that almost US\$100 trillion equivalent is traded annually on foreign exchange markets and that it is twenty times the value of world trade in goods.

## REFERENCES

- Baillie, R.T. and Selover, D.D. (1987) Cointegration and models of exchange rate determination, *International Journal of Forecasting*, **3**, 43–51.
- Charemza, W.W. and Deadman, D.F. (1992) *New Directions in Econometric Practice*, (Edward Elgar Publishing, UK).
- Conejo, C. and Shields, M.P. (1993) Relative PPP and the long run terms of trade for five Latin American countries: a cointegration approach, *Applied Economics*, **25**, 1511–15.
- Corbae, D. and Ouliaris, S. (1988) Cointegration and tests of purchasing power parity, *Review of Economics and Statistics*, **70**, 508–11.
- Dickey, D. A. and Fuller, W. A. (1981) Likelihood ratio statistics for auto-regressive time series with a unit root, *Econometrica*, **49**, 1057–72.
- Engle, R.F. and Yoo, B.S. (1987) Forecasting and testing in cointegrated systems, *Journal of Econometrics*, **35**, 143–59.
- Fuller, W.A. (1976) *Introduction to Statistical Time Series*, (J. Wiley, New York).

<sup>2</sup> Using precisely the same data set, the bilateral exchange rate and price ratio for New Zealand *vis-à-vis* Australia was subjected to precisely the same procedures reported above. Once again no evidence of cointegration was detected.

Holzer, S. (1992) Purchasing power parity: a guide for long term currency movements, *Swiss Bank Corporation Prospects*, **Aug/Sept**, 5–7.

McNown, R. and Wallace, M.S. (1989) National price levels, purchasing power parity and cointegration: a test of four high inflation economies, *Journal of International Money and Finance*, **8**, 533–45.

Rush, M. and Husted, S. (1985) Purchasing power parity in the long run, *Canadian Journal of Economics*, **18**, 137–45.

Yoonbai Kim (1990) Purchasing power parity in the long run: a cointegration approach, *Journal of Money, Credit and Banking*, **22**, 491–502.