What Fisher Knew About His Relation, We Sometimes Forget

A B S T R A C T

Empirical analyses of the Fisher relation overlook that expected consumption growth increases the real interest rate as consumers smooth consumption over time. This concept, which was understood by Fisher, is rooted in the representative agent’s optimization problem via the Euler equation governing bond purchases. Including consumption growth in the empirical analysis of the Fisher relation shows that the pro-cyclical variation in the ex-post real rate is largely explained by variation in expected consumption growth and that the coefficient estimates of expected inflation in the Fisher relation increases toward its expected value of 1. As a byproduct, this study also provides an alternative means of estimating the coefficient of relative risk aversion.

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and
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1. Introduction

The Fisher relation is probably one of the most studied relationships in macroeconomic theory. In the last two years alone there were more than a dozen studies on the Fisher relation, mostly featured in empirical journals. Fisher’s contribution was to observe that real and nominal factors can be separated when analyzing their effects upon the economy. He also recognized that the time shape of income is a determinative factor of the real interest rate.

“The fact that a person’s income is increasing tends to make his preference for present over future income high, as compared with what it would be if his income were flowing uniformly or at a slackening rate; for an increasing income means that the present income is relatively scarce and future income relatively abundant (Fisher 1930, p. 73-74).”

However, most studies of the Fisher relation have generally neglected to include expected real income, or more properly consumption growth as an explanatory variable. In this paper, we derive an augmented Fisher relation from a representative agent’s standard Euler equation governing the demand for bonds. The augmented form differs from the standard Fisher relation by the inclusion of an expected consumption growth term. We demonstrate that this inclusion is i) empirically significant, ii) helps reconcile the findings and resolves the puzzles of two seemingly exclusive branches of empirical study of the Fisher relation, and iii) provides an alternative way to estimate the consumer’s degree of relative risk aversion.

The few studies that have included measures of output growth as explanatory variables, e.g., Levi and Makin (1978), VanderHoff (1984), and Dotsey, Lantz and Scholl (2003) have not motivated the inclusion using Fisher’s argument found in the quote above and formalized in the model section of the paper. Levi and Makin argues that the Phillips curve introduces a ‘short run bias’ into the Fisher relation. They view this bias as disappearing in the long run.
because the economy reverts to the full employment level of output. Later, VanderHoff
includes the output gap, along with other variables, in a reduced form representation of the
real interest rate. Recently, Dotsey, Lantz and Scholl examine the ex-post real interest rate
and find that it is largely explained by contemporaneous output.

The remaining studies on the Fisher relation utilize its original form that excludes
consumption (or output) growth. Two somewhat related puzzles stand out in the literature.
The first one is that the observed ex-post real interest rate appears to be quite volatile, not in
line with the assumed fixed rate of time preference of consumers. Theoretical explanations
include Mundell (1963) and Tobin (1965) positing that variation in inflation over time affects
the demand and supply of bonds, respectively. Fama (1981) considers whether movements in
real factors, such as investment demand, over the business cycle are responsible for the
instability of the real rate. More recently, Evans and Lewis (1995) consider the case in which
inflation follows a Markov-switching process so that the ex-post observed real interest rate
features a deviation of inflation from its expected value, where this difference is time-varying
and serially correlated. The empirical papers have offered methodological and data issues as
the underlying reason for the problem of time variation in the ex-post real rate. Some of these
articles discuss whether the real rate is difference stationary (Rose, 1988; Mishkin 1992, 1995;
Sun and Phillips, 2004) while others claim that it simply follows a non-linear process in the
form of Markov switching process (Evans and Lewis, 1995; Garcia and Perron, 1996). We
contribute to the theory literature by deriving an ‘augmented Fisher relation’ that incorporates
consumption growth. In doing so, we not only help elaborate on Fisher’s comment, but also
provide a theoretical explanation for the correlation of the ex-post real rate with real output
growth.
While the first branch above calculates the ex-post real rate assuming that the coefficient on inflation expectations is constant at 1, the second branch focuses on estimating this coefficient assuming that the ex-ante real rate is constant. The observed response of the nominal interest rate to the expected inflation rate is routinely less than the (ex-ante) theoretical value of 1. A number of articles attempt to solve this puzzle by criticizing the data that has been used (Gilbert and Yeoward, 1994; Dotsey, Lantz and Scholl, 2003) while others suggest using alternative methodologies (Woodward, 1992; Crowder and Hoffman, 1996; Ng and Perron, 1997; Sun and Phillips, 2004; Caporale and Pittis, 2004). We argue that the omission of consumption growth puts a downward bias on the estimate of the inflation expectation coefficient and find that its inclusion reduces the bias. Our contribution shows that puzzles in the two seemingly unrelated branches of research may have a common solution. While illustrating that a large part of the variation in the real rate is due to expected consumption growth, we also show that its inclusion pushes the inflation expectation’s coefficient estimate closer to its theoretical value of 1.

The paper is organized as follows: Section 2 elaborates on why and how we should add the consumption growth in the Fisher relation. Section 3 empirically examines the theoretical arguments and interprets the results while Section 4 concludes.

2. Model

In this section we show that Fisher’s quote above is consistent with the consumer’s Euler equation for bond purchases. We begin with the Euler equation for an asset returning a fixed nominal rate of return, which is consistent with a number of primary sources including Lucas (1978), Hansen and Singleton (1982), Svensson (1985) as well as textbooks:
\[ \frac{u_c(c)}{p} = \beta (1 + \tilde{i}) E \left[ \frac{u_c(c')}{p'} \right] \]  

where a prime denotes variables measured at time \( t + 1 \), a tilde distinguishes the interest rate in levels from the log of one plus the interest rate used in some of the equations below, and \( E \) is the expectations operator at time \( t \). In Equation (1), the optimizing consumer equates the utility value of the last dollar spent on current consumption with the discounted expected future utility value of a dollar, saved now for consumption in the next period. If we assume a constant degree of relative risk aversion (CRRA) utility function:

\[ u(c) = \begin{cases} 
\frac{1}{1-\gamma}c^{1-\gamma} & \gamma \neq 1 \\
\ln c & \gamma = 1 
\end{cases} \]  

and rearrange terms, then Equation (1) becomes:

\[ (1 + \tilde{i})^{-1} = \beta E \left[ \frac{p}{p'} \left( \frac{c}{c'} \right)^\gamma \right] \]  

where \( \gamma \) represents the consumer’s degree of relative risk aversion. Expanding the expectation term we obtain:

\[ (1 + \tilde{i})^{-1} = \beta \left\{ E_i \left( \frac{p}{p'} \right) E_i \left( \frac{c}{c'} \right)^\gamma + \text{cov} \left[ \frac{p}{p'}, \left( \frac{c}{c'} \right)^\gamma \right] \right\} \]  

To simplify this expression we define the following terms:

\[ \rho = \ln \beta^{-1}, i = \ln (1 + \tilde{i}), \pi^e = -\ln E \left( \frac{p}{p'} \right), g^e = -\ln E \left( \frac{c}{c'} \right) \]
where $\rho$ represents the consumer’s rate of time preference, $i$ represents the nominal interest rate, $\pi^e$ is a measure of the expected inflation rate, and $g^e$ represents a measure of the expected consumption growth rate.\(^1\)

If the covariance term in Equation (4) is negligible as would be expected in a low inflation risk economy (as shown in Sarte 1998), then we obtain our augmented Fisher equation by taking the natural log of both sides of Equation (4) and substituting in the terms from Equation (5):

$$i = \rho + \pi^e + \gamma g^e$$ \hspace{1cm} (6)

Equation (6) indicates that the nominal interest rate must compensate the consumer according to his or her rate of time preference, the expected rate of decrease in purchasing power due to inflation, and for the utility cost of having non-smooth expected consumption.\(^2\) Compared to the original form of the Fisher equation, $i = r + \pi^e$, Equation (6) separates the effect of anticipated consumption growth from the rate of time preference in the determination of the real interest rate:

$$r = \rho + \gamma g^e$$ \hspace{1cm} (7)

Lucas (2003) refers Equation (7) to as ‘familiar’, and Laubach and Williams (2003) use this in a complementary study to illustrate the time variation in the ‘natural rate of interest’. Note

\(^1\) The last two items require further explanation. First, to be in conformance with the consumer’s Euler equation, the proper inflation measure is the ‘inverse of the expected rate of decrease in purchasing power due to inflation’ rather than the rate of inflation itself. According to Jensen’s inequality these are not the same in the presence of risk. The same concept applies to measuring the consumption growth rate. In this study we proxy for the measure of inflation expectations by a single point, namely the actual inflation, so there is no difference between $-\ln E(p/p')$ and $\ln E(p'/p)$.

\(^2\) A number of papers, Darby (1975), Feldstein (1976), Peek (1982), Crowder and Wohar (1999), to name a few, have argued that the Fisher equation should apply to the after-tax nominal interest rate (i.e., $(1 - \tau)i = \rho + \pi^e + \gamma g^e$), we opt to use the standard form because it is difficult to calculate the relevant marginal effective tax rate on T-Bill returns.
that Equation (6) provides us with an alternative method of estimating the coefficient of relative risk aversion, $\gamma$. There is already a long and varied empirical literature devoted to finding the degree of relative risk aversion. Recent additions include evidence from portfolio returns (Blake, 1996), health and retirement behavior (Barsky et al., 1997), hypothetical risks (Eisenhauer and Ventura, 2003), insurance purchase decisions (Halek and Eisenhauer, 2001), and gambling (Julien and Salanie, 2000).

3. Data and Estimation

Data

To test the theory we use US quarterly data for 1-year Treasury constant-maturity bonds (from the Board of Governors), seasonally-adjusted real personal consumption expenditures (from the Bureau of Economic Analysis), and the consumer price index for all urban consumers (from the Department of Labor) for the sample period of 1960Q4 to 2005Q4 (Figure 1). The choice of US data is compatible with our earlier assumption and Sarte’s (1998) finding that the second order term in Equation (4) is small. The inflation and consumption growth expectations are proxied by annual growth rates of CPI and consumption data, respectively (Sun and Phillips, 2004). The endogeneity problem that the proxies might cause will be addressed by an appropriate system estimation methodology. The annualized growth rates at quarterly frequency are obtained by a fourth differencing of the log-levels, in other words $\text{growth} = x_t - x_{t-4}$. The quarterly frequency of the annualized data is likely to cause a moving average process, which will also be removed prior to the final estimation. Finally, since the Treasury rates reflect the yields to be collected one year after purchase, we align

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3 The moving average problem that such differencing generates will be addressed shortly.
next year’s inflation and consumption growth rates with the current year’s interest rate (Sun and Phillips, 2004).

Estimation

The Fisher relation has been tested many times using a multitude of estimation techniques. Crowder and Hoffman (1996) and Caporale and Pittis (2004) show that Johansen’s cointegration test (1988, 1991) performs better than many other techniques in the testing of the Fisher relation, since it accounts for the endogeneity associated with replacing the inflation expectation with actual inflation. Although we also prefer system (VEC) estimation for the same reasons, we use the structural VAR model of Pesaran and Shin (henceforth PS, 2002) instead of Johansen’s methodology. The advantage of PS’s methodology is that it allows for a general framework for the identification, estimation, and hypothesis testing of the cointegrating relations without making prior restrictions on the number of common stochastic trends like the recursive method of Phillips (1991, 1995) or determining long run parameter values solely relying on statistical reduced rank regressions as in Johansen (1988, 1991). Using consistent QML estimators for the short and long run parameters, PS’s methodology enables one to test for general non-linear over-identifying restrictions on the cointegrating vectors, even across cointegrating equations. We elaborate on the advantages of PS approach in comparison to the other methodologies in the Appendix. Garratt et al. (2003) also applies PS’s technique to analyze the UK economy. Their paper incorporates long run relations in the

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4 System estimation, which proxies for expectations with actual data, imposes a learning process in expectations formation.
5 Due to the minimal restrictions we place on the long run structure of the model, our methodology is effectively very close to the Phillips technique (except for the maximization method) and also the results we obtain from the data dependent Johansen’s methodology are very close to the ones we get using PS’s method.
system, including the Fisher relation, among others, and it also includes an exogenous \(I(1)\) forcing variable, namely oil prices. Using their notation, the system can be represented as:

\[
\Delta y_t = a_y - \alpha_y \xi_t + \sum_{i=1}^{z-1} \Delta y_{t-i} + \sum_{j=0}^{z-1} \Psi_j \Delta z_{t-j} + u_y
\]

where \(y\) is the set of endogenous variables in the system, \(a_y\) is a vector of fixed intercepts, \(z\) is the set of exogenous (forcing) \(I(1)\) variables, \(\xi_t\) is the error correction term (that could include the forcing variable as well), and \(\alpha_y\) is the vector of loading coefficients. In our estimation we have \(y_t = (i_t, \pi_t)\) and the error correction terms are \(\xi_t = \theta' (y_t, g_t) - \rho\) where \(g_t\) is the consumption growth rate replacing the forcing variable \(z_t\), and \(\rho\) is the freely estimated real rate of time preference. We test for the above-mentioned theoretical model by checking the statistical difference of the cointegrating coefficient values from the theoretically dictated values of \(\theta = \begin{bmatrix} 1 & -1 \end{bmatrix} \{ -0.5 \text{ to } -2.5 \} \). Our range, 0.5 to 2.5, for the value of for the CRRA parameter, \(\gamma\), is consistent with what Kocherlakota (1996, page 52) refers to as “the current state of professional opinion” (see also Lucas, 2003, p. 6). Since the consumption growth is a forcing (exogenous) variable in our model, and testing its inclusion in the cointegrating vector requires short as well as long run (over-identifying) restrictions, PS’s methodology is quite compatible with our needs.

As an initial step, we test for the order of integration in the variables using the Modified Akaike Information Criterion (Ng and Perron, 2001) and find that they are all \(I(1)\).\(^6\) Next, we address the issue of a moving average component in the data due to the use of annualized

\(^6\) The degree of nonstationarity varies between the variables since we fail to reject the null of unit root at 95% for all series, but reject the null with 90% significance for consumption.
growth rates. To prevent distortion in our subsequent estimations, we clean the moving average component from the data by first estimating the MA coefficients of the $ARIMA(p,d,q)$ processes using the Exact Maximum Likelihood method and then use the inverted MA coefficients to eliminate the moving average component. The orders of $p$ and $q$ are selected using the Akaike model selection criterion and the truncation of the inverted MA coefficients is done when their levels are less than 0.05. We then take the cleansed data and utilize the PS’s (2002) methodology to test the long run relationship between the interest rate, inflation and consumption growth rates.

Table 1 displays the results of our estimation in which the lag length $s$ is chosen using the Akaike information criterion. From the results, one first notes that there is evidence of cointegration between interest and inflation rates in both cases. However, the coefficient estimates that one obtains from the two different specifications of the Fisher relation are quite different from one another. These significantly different results show that the addition of consumption belongs in the Fisher relation. The significant $t$–statistic is validated by the likelihood ratio’s strong rejection of the over-identifying (long and short run) restriction in the cointegrating vector. Its inclusion in the ‘augmented’ Fisher relation is nontrivial since its coefficient provides the practitioner with alternative means to measure relative risk aversion. The empirical estimate of 1.44 is quite consistent with micro and macroeconomic studies using different methodologies.

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7 The model specification for the interest rate is ARIMA(4,1,2) with MA coefficients 0.58 and -0.25; for inflation we find ARIMA(3,1,2) with MA coefficients 0.24 and -0.10; for consumption the specification is ARIMA(1,1,3) with MA estimates -0.21, 0.04 and 0.04.

8 Lag length is selected as 5 and 1 for the cases of with and without consumption growth, respectively. Changes in the lag length by 2 in either direction do not alter the results and the conclusions quantitatively or qualitatively.
In our opinion, omission of consumption growth is also responsible for some of the discussions surrounding the estimate of the inflation coefficient due to the bias it causes in the estimation of both the inflation coefficient and the real rate. Its inclusion provides us with another important outcome of our study, namely the increase in the coefficient estimate of inflation when compared to earlier studies like Mishkin (1992) and Evans and Lewis (1995). A number of earlier studies have investigated the underlying reason for the coefficient estimate of inflation being significantly below unity (Crowder and Hoffman, 1996; Ng and Perron, 1997; Caporale and Pittis, 2004) and in general have pointed to methodological issues as the culprit. We run estimations for the identical regression with and without the consumption growth to find that the coefficient estimates change dramatically. While the structural estimation of the standard Fisher equation results in an inflation coefficient of 0.56 that is significantly different than 1, our modified version has a higher coefficient estimate of 0.75 that is not statistically different from 1. Hence, the inclusion of consumption growth in the Fisher relation seems to move the estimate of inflation expectation coefficient in the right direction.

Another important result of the estimations concerns the real rate, which is rarely reported in the empirical literature. While the estimate for the consumer’s time preference rate (despite being large and having the wrong sign) is not significantly different than zero, the estimate for the real rate in the regression without consumption is estimated at 2.18%. We conclude that the significant difference between the time preference parameter in the augmented Fisher equation and the real interest rate in the model without consumption shows

\[\text{Applying a coefficient of 1 in the Fisher equation results in an ex-post real rate with a slight negative mean that is not significantly different than 0. The lack of a theoretical explanation for this ubiquitous finding may explain the lack of focus on the real interest rate.}\]

\[\text{Lucas (2003) argues that CRRA values larger than 2.5 should correspond to negative time preference rates.}\]
that most of the level and some of the fluctuations of the real rate are due to growth expectations.\footnote{This finding is consistent the theoretical findings of Arnwine (2006), which demonstrates that a real interest rate that is lower than the consumer’s rate of time preference occurs in an time-dynamic overlapping generations model.} Other specification tests show that we have heteroscedasticity in both specifications\footnote{The heteroscedasticity is due to the oil crises era in which the real rate tends to fluctuate wildly.} with residual serial correlation only in the standard Fisher form.

We run the identical regression using annual frequency data for robustness check. The results, displayed in Table 2, provide us with the confirmation of our earlier findings and another interesting outcome. The consumption growth variable is still very significant with even a larger CRRA value of 2.16. The coefficient of inflation is now even higher with a slightly larger standard deviation and the time preference is still not significantly different than zero. One interesting outcome of testing in annual frequency is the increase in the coefficient of inflation to 1 even without consumption growth. This phenomenon, characterized earlier by Levi and Makin (1978) as the economy reverting to the full employment level, is likely due to the short run fluctuations in consumption growth mattering less in making optimizing saving decisions and the real rate getting closer to its steady state constant value.

4. Conclusion

While it is accepted as fact that expected consumption growth affects the real interest rate, this has been overlooked in studies of the Fisher relation. We find that including a consumption growth term in the estimation of the Fisher equation 1) yields a significant coefficient for consumption growth, 2) allows us to estimate the consumer’s degree of relative risk aversion, 3) pushes the coefficient estimate for inflation expectation closer to its theoretical value of 1,
and finally 4) explains some of the instability in the real interest rate observed in previous studies. Therefore, we conclude that the future studies of the Fisher relation would benefit from including expected consumption as an explanatory variable.

Appendix

Using the notation of Pesaran and Shin (2002) for the coefficients we utilize a VEC model

$$\Delta y_t = a_y - \Pi y_{t-1} + \sum_{i=1}^{s-1} \Gamma_{yi} \Delta y_{t-i} + \sum_{i=0}^{s-1} \Psi_{zi} \Delta z_{t-i} + u_{yt}$$

where \( y \) is the set of endogenous variables in the system, \( a_y \) is a vector of deterministic trends, \( z_i \) is the set of exogenous (forcing) \( I(1) \) variables, \( \Pi = \alpha' \beta \), in which \( \alpha \) is the vector of loading coefficients and \( \beta \) is the cointegrating vector. In Johansen’s (1988, 1991) exactly identified estimator, the estimate for the cointegrating vector is obtained by the first \( r \) eigenvectors of \( B' - A' \) (or \( S_{10} S_{01}^{-1} S_{01} \) where \( S_y = T^{-1} \sum_{t=1}^{T} r_{it} r_{jt}' \), \( i, j = 0,1 \) and \( r_{0t} \) and \( r_{1t} \) are the residual vectors from the regressions of \( \Delta y_t \) and \( y_{t-1} \) on \( \{1, t, \Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t-p+1}\} \), respectively) with respect to \( B' \) or \( (S_{11}) \) subject to orthonormalization and orthogonalization restrictions \( \beta' B' \beta \) and \( \beta_i' (B' - A') \beta_j \) (\( i \) and \( j \) are columns). These conditions impose \( r^2 \) exact restrictions and provide a mathematically convenient way to identify the long run parameters.

The identification of Philips (1991) is based on an a priori decomposition of \( y_t \) into two sub-vectors, \( x_{1t} \) and \( x_{2t} \) where \( x_{2t} \) are not cointegrated among themselves. Under this decomposition, the number of cointegrating vectors is not estimated but known. The \( r^2 \)
restrictions are placed by setting the coefficients of \( x_{it} \) to the identity matrix. Hence, Phillips relies on a triangular characterization of the system while Johansen employs empirical identification restrictions.

Pesaran and Shin (2002) utilize QML estimation and derive the rank and order conditions for their long run structural method. Their order condition for the long run parameters requires that the number of nonlinear restrictions is greater or equal to the number of cointegrating relations squared, \( k \geq r^2 \) while the rank condition requires that there are \( r \) restrictions per each \( r \) vectors. The real advantage of this methodology, compared to the others, is when there is more than one cointegrating relationship and, possible, nonlinear relations between the coefficients of the system. Despite the fact that we have neither in our estimation, we utilize the Pesaran and Shin methodology to \( i \) test long and short run restrictions on the coefficients of the forcing \( I(1) \) variable, \( ii \) avoid an \textit{a priori} structuring of the model, as in Phillips (1991), and \( iii \) to incorporate more of the economic theory into the estimations, not solely depending on the data (as in Johansen, 1991) to set the structure of the model.
References


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### Table 1: Cointegrating Vector Estimates of the Fisher Equation (with and without consumption) (quarterly)

<table>
<thead>
<tr>
<th></th>
<th>T-Bill</th>
<th>Inflation</th>
<th>Consumption Growth</th>
<th>Time Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coefficient</strong></td>
<td>1.00</td>
<td>-0.75</td>
<td>-1.44**</td>
<td>6.68</td>
</tr>
<tr>
<td><strong>t-statistic</strong></td>
<td>0.83†</td>
<td></td>
<td>-2.18</td>
<td>1.53</td>
</tr>
<tr>
<td><strong>Cointegration Rank</strong></td>
<td></td>
<td>Max-Eigen</td>
<td>95% cv</td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td></td>
<td></td>
<td></td>
<td>29.10</td>
</tr>
<tr>
<td>r&lt;= 1</td>
<td></td>
<td></td>
<td></td>
<td>10.06</td>
</tr>
<tr>
<td>Serial Correlation (F test)</td>
<td></td>
<td>F(4,154) = 0.76[0.553]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity (F test)</td>
<td></td>
<td>F(1,170) = 17.81[0.000]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-Bar-Squared (for ECM)</td>
<td></td>
<td>$\overline{R}^2 = 0.27$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T-Bill</th>
<th>Inflation</th>
<th>Consumption Growth</th>
<th>Real Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coefficient</strong></td>
<td>1.00</td>
<td>-0.56**</td>
<td></td>
<td>-2.18**</td>
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<tr>
<td><strong>t-statistic</strong></td>
<td>3.67†</td>
<td></td>
<td></td>
<td>-4.04</td>
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<tr>
<td><strong>Cointegration Rank</strong></td>
<td></td>
<td>Max-Eigen</td>
<td>95% cv</td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td></td>
<td></td>
<td></td>
<td>38.67</td>
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<tr>
<td>r&lt;= 1</td>
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<td></td>
<td></td>
<td>8.70</td>
</tr>
<tr>
<td>Serial Correlation (F test)</td>
<td></td>
<td>F(12,494) = 6.5007[.000]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity (LM test)</td>
<td></td>
<td>F(1,520) = 27.6039[.000]</td>
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<td></td>
</tr>
<tr>
<td>R-Bar-Squared (for ECM)</td>
<td></td>
<td>$\overline{R}^2 = 0.14$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR test of over-identifying restriction</td>
<td></td>
<td>21.07 [0.000]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Lag length in the structural VAR model is selected as 6 using the Schwarz Information Criterion. **(*) indicates 95(90)% significance. † indicates that the coefficient is tested for its significant difference from 1. The difference between the two sections of results is the exclusion of the consumption growth variable. Without the consumption growth variable, the estimated constant represents the real rate. The LM tests have the nulls of no serial correlation and no heteroskedasticity. Lag length is selected as 5 and 1 for the cases of with and without consumption growth, respectively.
Table 2: Cointegrating Vector Estimates of the Fisher Equation (with and without consumption) (annual)

<table>
<thead>
<tr>
<th></th>
<th>T-Bill</th>
<th>Inflation</th>
<th>Consumption Growth</th>
<th>Time Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>1.00</td>
<td>-1.03</td>
<td>-2.16**</td>
<td>5.69</td>
</tr>
<tr>
<td>t-statistic</td>
<td>0.12†</td>
<td></td>
<td></td>
<td>1.68</td>
</tr>
</tbody>
</table>

Cointegration Rank

- Max-Eigen 95% cv
- $r = 0$ 22.60 18.88
- $r<= 1$ 5.28 12.45

Serial Correlation (F test) $F(1,30) = 0.06 [0.814]

Heteroscedasticity (F test) $F(1,39) = 3.24 [0.080]

R-Bar-Squared (for ECM) $\bar{R}^2 = 0.30$

Notes: Lag length in the structural VAR model is selected as 6 using the Schwarz Information Criterion. **(*) indicates 95(90)% significance. † indicates that the coefficient is tested for its significant difference from 1. The difference between the two sections of results is the exclusion of the consumption growth variable. Without the consumption growth variable, the estimated constant represents the real rate. The LM tests have the nulls of no serial correlation and no heteroskedasticity. Lag length is selected as 4 and 1 for the cases of with and without consumption growth, respectively.
Figure 1: Components of Augmented Fisher Relation